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Discussion of Morris and Shin's "Rethinking Multiple Equilibria in Macroeconomic Modelling"

Andrew Atkeson

Federal Reserve Bank of Minneapolis and University of Minnesota

Macroeconomists have used coordination games with multiple equilibria to describe any number of phenomena in which we appear see large changes in economic outcomes with little or no apparent change in the underlying economic fundamentals. Usually, in macroeconomic applications, these games are shown to have multiple equilibria and the argument is made that large changes in economic outcomes can follow from changes in agents' expectations about what other agents will do rather than from changes in economic fundamentals alone.

Morris and Shin present a simple and dramatic insight into the structure of simple coordination games. With only a few assumptions, they show that if agents see a noisy signal of the true state of the world and thus have some uncertainty about the exact structure of the coordination game that they are playing as well as some uncertainty about what every other agent knows about the coordination game that they are playing, then these games in fact have a unique equilibrium corresponding to each underlying state of the world. This result suggest that Macroeconomists should reassess whether their previous findings of multiple equilibria in these coordination games is robust to small changes in the structure of information available to agents.

Morris and Shin go on to show that if the noise introduced into the coordination game is small, the selected equilibrium has the feature that there is a threshold state of the world around which the economic outcome changes very rapidly with small changes in the state, while in the other regions of the state space, the economic outcome is quite stable as the underlying state of the world varies. This second feature of the equilibrium selected by the Morris and Shin apparatus suggests that their work may be more than a criticism of the robustness of previous multiple equilibrium literature and may, in fact, have important implications for a number of applications.

My discussion of this paper by Morris and Shin has four parts. First, I present what I think is the simplest environment in which to apply the Morris and Shin apparatus. Second, I go over a proof of their result in this environment that is slightly different than the proof presented in the paper. I hope that it will help any reader interested in understanding the logic of Morris and Shin's results to see the argument from a different angle. Third, I describe what I find to be the most interesting feature of the equilibrium that is selected and also go over an example of how one might use this their technology to do comparative statics. Fourth, I describe what I believe is the main impediment to the use of this Morris and Shin technology for modelling macroeconomic phenomenon.

To jump ahead for a moment, this fourth and final part of my discussion does not focus on the applied question of whether the models that Morris and Shin have proposed for currency crises and the pricing of corporate debt in related papers are relevant for analyzing those phenomenon. Instead I focuses on the broader question of whether one can introduce markets and prices, clearly essential parts of any macroeconomic application, into what, to date, has been a purely game theoretic analysis. Morris and Shin, in their introduction, criticize previous applications of coordination games in macroeconomics for relying on assumptions that "allow agents actions and beliefs to be perfectly coordinated in a way that invites multiplicity of equilibria". The noise that they introduce into coordination games has the effect of preventing coordination of agents' actions and beliefs. In a market economy, however, prices serve precisely to coordinate actions (so that supply equals demand), and in a dynamic market economy, asset prices play an important role in coordinating agents' beliefs since these prices tend to aggregate information across individuals.

It is not clear to me how the argument presented by Morris and Shin would carry over to a model with markets. Their arguments require agents to have diverse beliefs about the probabilities of future outcomes in equilibrium and this typically does not happen in models in which agents see the market signals about those probabilities embodied in asset prices. The nature of this difficulty in translating the Morris and Shin technology to a market environment should become clearer after we review the details of how this technology works.

1. A simple coordination game

Let us review how this Morris and Shin technology works in the context of what seems like a natural application of the game theory. Consider a crowd that faces riot police in the street. Individuals in the crowd must decide whether to riot or not. If enough people riot, the riot police are overwhelmed, and each rioter gets loot $W > 0$. If too few people riot, the riot police contain the riot, and each rioter gets arrested with payoff $L < 0$. Individuals who choose not to riot leave the crowd and get safe payoff 0. The strength of the riot police depends on the state of the world θ, where the strictly increasing function $a(\theta)$ indexes the fraction of the crowd that must riot to overwhelm the police. Let $\underline{\theta}$ denote the point at which $a(\theta)$ crosses 0 and $\bar{\theta}$ the point at which $a(\theta)$ crosses 1.

The equilibria of this game when the state θ is common knowledge are as follows. If $\theta \leq \underline{\theta}$, then it is a dominant strategy for each individual to riot, since the riot police in this case are so weak that they cannot stop even a single rioter ($a(\theta) \leq 0$). Thus, if θ is in this region of the state space, everyone riots and gets payoff W for sure. If $\theta > \bar{\theta}$, then it is a dominant strategy for each individual not to riot since the police can contain the crowd even if everyone riots ($a(\theta) > 1$). Thus, if θ is in this region of the state space, no one riots and everyone gets payoff 0 for sure. In the middle of the state space, with $\underline{\theta} < \theta \leq \bar{\theta}$, there are two possible equilibria corresponding to each value of the state θ. In the first of these equilibria, everybody riots. In this case, the fraction of the crowd that riots is $1 \geq a(\theta)$, so the police are overwhelmed and everybody gets payoff $W > 0$. In the second of these equilibria, nobody riots. In this case the fraction of the crowd that riots is $0 < a(\theta)$, so the police contain the crowd and any individual who riots is arrested. Hence, nobody riots and everybody in the crowd gets $0 > L$. This game clearly has multiple equilibria in a region of the state space, and when the state variable is in this region, the economic outcome depends on agents' expectations of what other agents will do and not on the underlying economic fundamental $a(\theta)$.

2. An alternative presentation of the proof of their result

Morris and Shin introduce the following changes to this coordination game. They assume that individuals do not know the state of the world θ. Instead, each individual starts with a common prior that θ is normally distributed with some mean m_θ and variance $1/\alpha$ (precision α). (I think of the randomness in θ as arising from the problem that the precise strength of the squad of riot police available to face any particular crowd in any particular street at any particular time depends somewhat on chance). Each individual in the crowd then receives an idiosyncratic signal $x_i = \theta + \varepsilon_i$, of the state θ, where ε_i is normally distributed with mean 0 and variance $1/\beta$ (precision β) and is i.i.d. across individuals. Given these assumptions, we have two distributions that play a key role in the analysis. First is the distribution of signals x_i across agents conditional on the realization of the state θ. With the assumptions above, this is a normal distribution, but we can write in more generally as a c.d.f. $Prob(x \leq x^*|\theta)$, which we will assume to be a strictly positive, continuous, decreasing function of θ for any value of x^*. Second is the posterior distribution over θ for an agent who has seen signal x. This is obtained from Bayes' rule, and, under the assumptions above is a normal distribution, but can also be written more generally as a c.d.f. $Prob(\theta \leq \theta^*|x)$. We also assume that this is a continuous and decreasing function of x for any value of θ^*.

The Morris and Shin result in the context of this simple game can then be stated as follows. Assume that there is a unique solution x^*, θ^* to the following two equations

(1) $$Prob(x \leq x^*|\theta^*) = a(\theta^*),$$

(2) $$Prob(\theta \leq \theta^*|x^*)W + [1 - Prob(\theta \leq \theta^*|x^*)]\, L = 0.$$

Then there is unique equilibrium described by x^* and θ^*. The signal x^* is a threshold signal such that all individuals who get signals $x \leq x^*$ riot, and those who get signal $x > x^*$ do not riot. The state θ^* is a threshold state such that the crowd overwhelms the police and rioters get payoff W if $\theta \leq \theta^*$ and the police contain the crowd and rioters are arrested and get payoff L if $\theta > \theta^*$.

(In the paper, Morris and Shin make assumptions on the precision of the signal relative to the precision of the prior in stating the result. In proving their proposition they show that there is a unique solution to the analogs to equations (1) and (2) if we assume that the precision of the signal, denoted β, is sufficiently high relative to the precision of the prior, denoted α, and the slope of the function $a(\theta)$. The necessary and sufficient condition for their result, however, appears to be that these two equations have a unique solution).

One way to prove this proposition is by iterated deletion of dominated strategies. I find this proof of their proposition the easiest to understand. This iterated deletion of dominated strategies goes as follows.

First observe that individuals who get sufficiently low and high signals, which I denote x_0 for the low signal and x^0 for the high signal, are so confident in their posterior beliefs that $\theta \leq \underline{\theta}$ or $\theta > \bar{\theta}$, that they find it a dominant strategy to riot or not riot respectively regardless of what everyone else does. The low signal x_0 is the highest value of x such that

$$Prob(\theta \leq \underline{\theta}|x)W + Prob(\theta > \underline{\theta}|x)L \geq 0.$$

The interpretation here is that, even if one believed that everyone else in the crowd was not going to riot, and thus any individual rioter would be arrested in the event that $\theta > \underline{\theta}$, the posterior probability that $\theta \leq \underline{\theta}$ for someone who saw $x \leq x_0$ is high enough to make it worthwhile to run the risk of rioting.

Analogous reasoning defines x^0. Even with the belief that everyone else always riots, and thus the belief that rioters will get W if $\theta \leq \bar{\theta}$, someone who saw signal $x > x^0$, where x^0 is the smallest x such that

$$Prob(\theta \leq \bar{\theta}|x)W + Prob(\theta > \bar{\theta}|x)L \leq 0$$

would not find the potential reward of rioting likely enough to justify the risk. These two observations gives us the first round of deletion of dominated strategies: any equilibrium strategy must have all agents with signals $x \leq x_0$ rioting and those with signals $x > x^0$ not rioting because, for agents with such signals, rioting and not rioting are optimal strategies regardless of what everyone else does.

In the subsequent rounds of our iterated deletion of dominated strategies, we take as given the restriction on dominated strategies obtained from the previous round. That is, any individual contemplating the actions of others must believe that everyone who has signals $x \leq x_0$ will riot and signals $x > x^0$ will not riot. If everyone who has signals $x \leq x_0$ riots, then the fraction of the crowd that riots in state θ must be at least $Prob(x \leq x_0|\theta)$. Given our assumptions on this c.d.f., this fraction of rioters is always positive and is a continuous and declining function of θ. Thus, there is a maximum value of the state, which I denote $\theta_0 > \underline{\theta}$, such that

$$Prob(x \leq x_0|\theta) \geq a(\theta).$$

Accordingly, a rational individual must realize that at least in all states of nature $\theta \leq \theta_0$, enough of the crowd will riot to overwhelm the police, and such an individual thus finds it a dominant strategy to riot as long as his signal $x \leq x_1$, where x_1 is the largest signal x such that

$$Prob(\theta \leq \theta_0|x)W + Prob(\theta > \theta_0|x)L \geq 0.$$

Likewise, each agent realizes that at least fraction $Prob(x > x^0|\theta)$ of the crowd will not riot in state θ, and thus the rioters must lose and be arrested in all states greater than or equal to θ^0 where $\theta^0 < \bar{\theta}$ is the maximum value of θ such that

$$Prob(x \leq x^0|\theta) \geq a(\theta).$$

Accordingly, it is a dominant strategy for a rational agent not to riot when his signal exceeds x^1, where x^1 is the smallest x such that

$$Prob(\theta \leq \theta^0|x)W + Prob(\theta > \theta^0|x)L \leq 0.$$

With these observations we iteratively delete dominated strategies: given x_0 and x^0 as threshold signals below which everyone riots and above which no one riots, we have shown that any equilibrium strategy must have the crowd winning at least in states $\theta \leq \theta_0$ and losing at least in states $\theta > \theta^0$ and thus rational agents should riot when their signals $x \leq x_1$ and not rioting when their signals $x > x^1$. These new threshold signals x_1 and x^1 then take the place of x_0 and x^0 as restrictions on the behavior of every other agent and we go through these calculations again deriving new restrictions on the equilibrium strategies.

This iterative procedure of restricting the equilibrium strategies defines increasing sequences $\{x_n, \theta_n\}_{n=0}^{\infty}$ and decreasing sequences $\{x^n, \theta^n\}_{n=0}^{\infty}$ that progressively put tighter and tighter bounds on the equilibrium strategies. To finish the proof of Morris and Shin's proposition, we need only show that these sequences converge to common limit points, which I will denote x^* and θ^*. Showing this proves the proposition because it forces the conclusion that all agents with signals $x \leq x^*$ riot, while all agents with signals $x > x^*$ do not riot, and that the crowd wins the riot in all states $\theta \leq \theta^*$, and loses in all states $\theta > \theta^*$.

To show that the sequences above have common limit points, we observe that any limit points x^* and θ^* of either of these two sequences must be a solution to the two equations (1) and (2). But, if these two equations have a unique solution, then we are done since that forces the conclusion that these two sequences have a common limit point.

The algebra behind Morris and Shin's result that equations (1) and (2) have a unique solution when the signals x are precise relative to the prior is straightforward. To do the algebra under the assumption of normality, observe that the term

$$Prob(x \leq x^*|\theta^*) = \Phi\left(\sqrt{\beta}\,(x^* - \theta^*)\right),$$

where Φ is a standard normal c.d.f., and use the fact that an agent who sees signal x has posterior over θ that is normal with mean $(\alpha m_\theta + \beta x) / (\alpha + \beta)$ and precision $(\alpha + \beta)$, to get that the term

$$Prob(\theta \leq \theta^*|x^*) = \Phi\left(\sqrt{\alpha+\beta}\left(\theta^* - \frac{\alpha m_\theta + \beta x^*}{\alpha + \beta}\right)\right).$$

Use equation (2) to get

$$(3) \qquad x^* = \frac{\alpha+\beta}{\beta}\theta^* - \frac{\alpha}{\beta}m_\theta - \frac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}\left(\frac{-L}{W-L}\right)$$

and plug this into (1) to get one equation in the threshold state θ^*

$$(4) \qquad \Phi\left(\frac{\alpha}{\sqrt{\beta}}(\theta^* - m_\theta) - \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}\left(\frac{-L}{W-L}\right)\right) = a(\theta^*).$$

Equation (3) gives us the threshold signal x^* at which an agent is indifferent between rioting and not given threshold state θ^* and the left-hand side of equation (4) gives us the fraction of the crowd who receive signals less than or equal to x^*. Any solutions to this equation (4) must lie in the interval $\left[\underline{\theta}, \bar{\theta}\right]$. Both sides of this equation are increasing functions: the left hand side looks like a normal c.d.f. with steepness determined by $\alpha/\sqrt{\beta}$ and the right hand side has whatever slope is assumed to reflect how the strength of the police varies with the state. The fact that there is at most one solution when β is large relative to α follows from the fact that the left-hand side becomes flat in θ over the interval $\left[\underline{\theta}, \bar{\theta}\right]$ in the limit as $\alpha/\sqrt{\beta}$ goes to zero. Note that if $\alpha/\sqrt{\beta}$ is large, then this equation typically has three solutions in the interval $\left[\underline{\theta}, \bar{\theta}\right]$ (since the c.d.f. looks more like an S over this interval) and thus the iterated deletion of dominated strategies does not pin down a unique equilibrium.

3. The selected equilibrium and comparative statics

Consider now what the unique equilibrium outcome looks like as a function of the state of nature θ. Note first that, whatever the state of nature θ, some portion of the crowd is going to riot and some portion of the crowd will not riot. All that varies with the state θ is the size of the fraction of the crowd that riots and whether the rioters overwhelm the police or are arrested.

The fraction of the crowd that riots in state θ is $Prob(x \leq x^*|\theta) = \Phi\left(\sqrt{\beta}\,(x^* - \theta)\right)$. This fraction, as a function of the state θ, is one minus a normal c.d.f. and thus looks like a reverse S curve. If the noise ε has a small variance, then this fraction begins to look like a step function: close to 1 for $\theta < x^*$ and close to 0 for $\theta > x^*$ with a steep transition from high to low as θ crosses the threshold signal x^*. Thus, the equilibrium relationship between the actions of the crowd and the strength of the police is highly non-linear. For large ranges of values of the state θ, we have changes in the strength of the police, $a(\theta)$, but little or no change in the fraction of the crowd that riots. On the other hand, for values of θ close to the threshold signal x^*, we have a large and sudden change in the number of people rioting. This is a very interesting result since it suggests that sudden shifts in agents' expectations with small changes in the state may play an important role in determining equilibrium outcomes despite the fact that the equilibrium is unique.

Now let us go through an example of how to use this technology to do comparative statics. The natural exercise in this example is to ask what effect changes in the average strength of the police (parameterized by m_θ) have on the equilibrium incidence of riots, computed as the ex-ante probability that the state θ is below the threshold θ^* or $\Phi\left(\sqrt{\alpha}\,(\theta^* - m_\theta)\right)$. Differentiating equation (4) gives us the result that as long as the left-hand side of (4) is flatter in θ than the right hand side (the same condition that ensures uniqueness of the solution), then $d\theta^*/dm_\theta < 0$. What this implies, of course, is that strengthening the police has two beneficial effects: first, it lowers the probability that the crowd will win the riot holding fixed the threshold state θ^*, and second, it leads to a reduction in the threshold state θ^* further reducing the probability that the crowd will overwhelm the police. Morris and Shin play up this shift in the threshold state in their application of this technology to the pricing of corporate debt. Note, of course, that this second effect, this shifting of the threshold state, is smaller the larger is β relative to α. In the limit, as $\alpha/\sqrt{\beta}$ goes to zero, this second effect disappears.

4. The problem with introducing prices into the model

So far in our analysis, individuals in this crowd have no information other than their own signal to consider when they decide whether to riot or not. This would be different, of course, if we introduced markets and prices into the model. Imagine, for the sake of this discussion, that individuals also could see asset prices and assume specifically that there is a traded asset with payout contingent on the claims that the insurance company that covers the property threatened by the rioters must pay. For simplicity, assume that the claims that the insurance company would have to pay following a riot take on only two values: a large value in the event that the crowd overwhelms the police and a small value in the event that the police keep the crowd under control. Imagine, as well, that assets trades continuously, so that individuals in the crowd can see asset prices after θ is realized but before they need to decide whether to riot.

On the one hand, if this asset ends up being priced in equilibrium in a way that accurately reflects its subsequent payout, it will have one price in all states $\theta \leq \theta^*$ (reflecting that the insurance claims will be large) and another price in all states $\theta > \theta^*$. (reflecting that the insurance claims will be 0). This, of course, will be a problem for our previous analysis. Every individual should be able to look at this asset price and know whether the crowd is going to overwhelm the police or not. Depending on the price, then, either every individual should strictly prefer to riot or to not riot. Agents' actions and beliefs would be coordinated since there would be no reason for any individual to act differently on the basis of his own signal. The logic of the Morris and Shin argument goes out the window.

On the other hand, if this asset does not get priced in equilibrium in a way that allows agents to infer whether the crowd will overwhelm the police or not, we must ask why it is not priced this way? How do we set up the model so that the asset price does not aggregate the information that all of the individuals in the economy have and thus reveal the true state?

The idea that individuals in a crowd considering whether to riot or not would consult asset prices via the newspaper or their handy wireless internet connections seems farfetched. That, in part, was my motivation for picking this example for my discussion. The analysis of Morris and Shin seems like it might work pretty well for this example. In the macroeconomic examples that Morris and Shin point to in their paper, however, asset prices are clearly a necessary part of the picture, and it is not at all clear how their arguments apply.

In Morris and Shin's example regarding speculative attacks on currencies, one would think that forward exchange rates (interest rate differentials) and options on exchange rates are readily observed by all market participants when they consider whether to attack or not. Their example in their earlier AER paper, like the riot example above, has agents holding diverse beliefs about the probability that the currency will be devalued and deciding whether or not to attack on the basis of those beliefs. But, if, given the fundamentals today, the equilibrium uniquely pins down whether the currency will soon be devalued or not, then it seems that these interest rate differentials and exchange rate option prices should reflect today which of these two outcomes will occur. If these prices do accurately reflect which outcome will occur, agents should coordinate their decision to attack or not based on these prices: everyone should attack if these asset prices indicate a devaluation will occur and no one should attack if these asset prices indicate that a devaluation will not occur. It does not make sense in this application to assume that agents will take different actions (attacking or not) on the basis of their private signals if publicly observed asset prices accurately reveal which outcome will actually occur. It thus does not make sense to apply the argument proposed by Morris and Shin to the analysis of currency attacks unless we can tell some story as to why interest rate differentials and exchange rate options do not reveal an immanent devaluation of the currency even if that devaluation must occur in equilibrium with probability one.

In Morris and Shin's example regarding corporate debt discussed in detail in a cited working paper, the price of the firm's equity and the secondary market price of the firm's debt will clearly reflect some market assessment of the likelihood that the firm will be liquidated in equilibrium. If the outcome, liquidation or not, is uniquely pinned down by the fundamentals, then these prices should reveal that and agents should be able to coordinate their actions accordingly.

Finally, in the bank run example presented in this paper, the price of the bank's equity should reveal whether there will be a run or not since this outcome is pinned down in equilibrium. Accordingly, agents should look at this price in deciding whether to run or not and it seems natural to suspect that their actions and beliefs might be coordinated upon the observation of this price.

The question then stands, how do we integrate prices into the analysis and yet preserve the diversity of posterior beliefs across agents that is key to pinning down a unique equilibrium? Perhaps the answer to this question will depend on the specific application: it seems plausible that rioters are not integrating asset prices into their analysis of whether to riot or not, it seems less plausible to assume that currency traders are ignoring interest rate differentials and option prices in deciding whether to attack a currency or not. Finding an answer to this question seems to me to be the obvious next step in refining this potentially useful technology for analyzing macroeconomic coordination games.